**Assignment for the Modal Logic component**

**of Overview of Logic COMP 6463**

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**1. Let be a Kripke model and let . Use the semantic clauses for the definition of around Slide 9 to prove that if then .**

**Answer:**

Assume that

and

i.e.

i.e.

Exist has and

And

i.e.

i.e.

(contradiction)

Thus, if then .

**2. Show a model such that . You must give concrete values of your , and as in my lecture notes. A diagram alone does not count but can be used to clarify your concrete values.**

**Answer:**

Then,

and , hence,

This model is shown in diagram below:

**3. Prove that a frame validates the shape if and only if the frame is dense i.e.**

**Answer:**

**Proof(i):**

Assume R is dense and for some

Exist model and with and

i.e. with

For R is dense, , hence and . Contradiction.

**Proof(ii):**

Assume forces all instances of , and R is not dense.

and there is no that holds.

Let ( it means , ), then

For forces all instances of , and is an instance of it,

, then . with

i.e.

i.e. holds. Contradiction.

Hence, a frame validates the shape if and only if the frame is dense i.e.

**4. Use the tableau method for K with global assumptions to decide whether the formula**

**is a logical consequence of . Show the tableau(x).**

**Answer:**

**(**

i.e. doesn`t have a closed K-tableau.

Hence, is not a logical consequence of **.**

**5. Give a tense logic model which satisfies You have to give a concrete , and . A diagram alone does not count but can be used to clarify your concrete values.**

**Answer:**

Then

Hence,

This model is shown in diagram below:

**6. Prove that the following formula is -valid.**

**Answer:**

Assume that, there is a model and some such that:

.

i.e.

i.e. and

i.e. has to have at least one successor and .

i.e. ,., with that

i.e.,., with that . Contradiction.

Hence, is -valid.

**7. Use the tableau method from the lecture notes for PLTL to decide whether the formula is PLTL-valid. Show the tableau.**

**Pass 1:**

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**Pass 1: Delete all closed nodes**

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**Pass 2: Delete any node which contains an unfulfilled eventuality**

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**Pass 2: Delete nodes with no children.**

All nodes are deleted step by step.

Root deleted so the formula is unsatisfiable.

Hence, the formula is PLTL-valid.

**8. Show that is true in all PDL-models.**

**Answer:**

:=

=

=

=

Hence, **.**

**9. Is the formula PDL-valid? Give your reasoning.**

**Answer:**

The formula is not PDL-valid. But we don`t know at the beginning, so I still assume it is valid.

For a contradiction, assume and in some PDL model

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:=

, assume that , then , that is,

Assume that is the only one state with

i.e.

We define a set S :={ }

i.e.

=

= }

=

:=

Thus, there exist some state in some PDL model , and without any contradiction.

Hence, the formula is not PDL-valid.

**10. Let us call a tense-logical formula “purely modal” if it contains no occurrences of nor . Prove the following theorem: a purely modal formula is -valid if and only if it is -valid.**

**Answer:**

is -valid means

i.e. because of the soundness and completeness of and the same to .

Hence, what we need to prove is that iff

**Proof(a):** if then

We can prove by induction on the length of the derivation of

**:** So because is an axiom schema instance of (can`t be derived by rule Id because contains no formula.).

Then must be an instance of .

i.e.

i.e. must be an instance of in

Thus, is an axiom schema instance of , hence

**Ind.Hyp:**Theorem holds for all derivations of length less than some .

**Ind.Step:** Suppose has a derivation of length k.

**MP:** So both and are shorter than . By IH and . Hence,

**Nec[F]: Suppose []=** then are is shorter than By IH .Hence,, that is,

**Proof(b):** if then

We can prove by induction on the length of the derivation of

**:** So because is an axiom schema instance of (can`t be derived by rule Id because contains no formula.).

Then must be an instance of .

i.e.

i.e. must be an instance of in

Thus, is an axiom schema instance of , hence

**Ind.Hyp:**Theorem holds for all derivations of length less than some .

**Ind.Step:** Suppose has a derivation of length k.

**MP:** So both and are shorter than . By IH and . Hence,

**Nec:** Suppose **[]=** then are is shorter than By IH .Hence,, that is,

From the above, iff

i.e. iff

i.e. a purely modal formula is -valid if and only if it is -valid.